

Static supersymmetric black holes in AdS_4 with spherical symmetry

Kiril Hristov^{*,†}, Stefan Vandoren^{*}

^{} Institute for Theoretical Physics and Spinoza Institute,
Utrecht University, 3508 TD Utrecht, The Netherlands*

[†] Faculty of Physics, Sofia University, Sofia 1164, Bulgaria

K.P.Hristov, S.J.G.Vandoren@uu.nl

Abstract

We elaborate further on the static supersymmetric AdS_4 black holes found in [1], investigating thoroughly the BPS constraints for spherical symmetry in $N = 2$ gauged supergravity in the presence of Fayet-Iliopoulos terms. We find Killing spinors that preserve two of the original eight supercharges and investigate the conditions for genuine black holes free of naked singularities. The existence of a horizon is intimately related with the requirement that the scalars are not constant, but given in terms of harmonic functions in analogy to the attractor flow in ungauged supergravity. The black hole charges depend on the choice of the electromagnetic gauging, with only magnetic charges for purely electric gaugings. Finally we show how these black holes can be embedded in $N = 8$ supergravity and thus in M-theory.

1 Introduction

The study of black holes in supergravity and string theory has been of general interest for many years. Research topics range from fundamental aspects of quantum gravity and microscopic state counting in string theory, to applications of black hole thermodynamics in strongly coupled field theories via the AdS/CFT correspondence. Many properties of black holes depend on the asymptotic spacetime they live in, which can be flat, de Sitter,

or anti-de Sitter (AdS). Most studies focus on asymptotically flat or AdS spacetimes, and in this work we focus on the latter.

In this paper we analyze a class of static supersymmetric (BPS), asymptotically AdS_4 black holes with a spherical horizon in gauged $N = 2$ supergravity. Static BPS solutions with other horizon topologies, as well as stationary rotating solutions, are known to exist for long time in such theories [2, 3]¹. However, until recently static BPS black holes with spherical horizons were thought not to exist, at least not for the choices of gauging and Killing spinor ansätze studied in e.g. [4]. While this is the case in minimally gauged supergravity with a bare cosmological constant [5], the first example of proper static BPS solution in the presence of vector multiplets and a scalar potential was derived in [1], building on earlier work [6].

Just like in [1], we concentrate on gaugings with Fayet-Iliopoulos (FI) terms. We do not consider hypermultiplets, but in certain cases the hypermultiplet gaugings allow for truncations to the models we consider here [7]. As we will explain, the FI terms determine the electric charges of the gravitini and are subject to a Dirac quantization in the presence of any magnetic charges. The black holes we study in this paper are magnetically charged, and have an entropy that depends on both magnetic charges and FI terms. The fact that they are quantized will therefore be important for the microscopic state counting.

The complete set of BPS conditions were written down in [1], with no constraint on the topology of the horizon and no assumption on the form of the Killing spinors. While this covers the most general case, the equations are somewhat cumbersome and difficult to analyze unless one specifies to detailed examples. Here, we aim to understand better the case of spherical horizons only, for which the BPS conditions simplify once we restrict to a particular class of Killing spinors. In this way, one recovers attractor-like equations that are similar to the ones describing asymptotically flat black holes in ungauged supergravity [8, 9, 10]. We also extend the analysis beyond the standard electrically gauged $N = 2$ supergravity, by allowing magnetic gaugings. In such models, we can describe more general black hole solutions that have both electric and magnetic charges on equal footing.

As an illustration, we consider the case of one vectormultiplet. This example was also studied in [1], where a spherically symmetric black hole with no naked singularity was found. We discuss further the properties of this black hole, such as the entropy formula and the attractor mechanism. Furthermore, we also comment on the mass of the black

¹Note that, unlike the case for asymptotically flat static black holes, the topology of the horizon of AdS_4 black holes is not unique. The horizon can be a Riemann surface of any genus as explained in [2].

hole and describe the embedding into eleven-dimensional supergravity.

The plan of the paper is as follows. First, in section 2 we discuss the known static AdS black holes in four dimensions and explain how the solutions described in this paper fit in the general picture. In section 3 we briefly outline some details about gauged $N = 2$ supergravity and explain our notations. In section 4 we specify in full detail our assumptions for spacetime and gauge fields and make a particular ansatz for the Killing spinors in order to simplify the BPS conditions. In section 5 we show how to solve the equations for the metric and scalar fields in terms of harmonic functions. We then proceed in section 6 to explain how the embedding tensor formalism [11] restores electromagnetic duality and propose a more general solution in an arbitrary electromagnetic frame. In section 7 we give some explicit examples of prepotentials leading to black hole solutions and give more details about the physical properties of these black holes and the attractor flow. In the last part of the paper, we show in section 8 how one can embed these new black holes in $D = 4$ $N = 8$ supergravity and in M-theory, thus suggesting a way to study their microscopic origin. In section 9 we comment on the mass of the black holes and their behavior in the large charge limit, which shows some quite unusual and puzzling behavior. We conclude with some further remarks and suggestions for future study. Some details about our gamma matrix conventions are left for the Appendix.

Note added: Just before this paper was submitted, we received reference [30] that has some overlap with our results in the sections discussing the dyonic solutions with electromagnetic gauging and the attractor mechanism.

2 Static AdS black holes

We focus on static spherically symmetric spacetimes with metrics of the form (the signature is $(+, -, -, -)$ in our conventions)

$$ds^2 = U^2(r) dt^2 - U^{-2}(r) dr^2 - h^2(r) (d\theta^2 + \sin^2 \theta d\varphi^2) , \quad (2.1)$$

for some functions $U(r)$ and $h(r)$ to be determined from the BPS conditions and/or the equations of motion.

For Minkowski spacetime, we have $U = 1$ and $h = r$, and for four-dimensional anti-de Sitter spacetime, one has

$$AdS_4 : \quad U^2(r) = 1 + g^2 r^2 , \quad h(r) = r , \quad (2.2)$$

where g is related to the cosmological constant of AdS_4 through the scalar curvature relation $R = -12g^2$. So, in the standard conventions the cosmological constant is $\Lambda = -3g^2$. For the Reissner-Nordström black hole solution in AdS_4 (RN-AdS), with mass M and electric and magnetic charges Q and P , we have

$$\text{RN} - \text{AdS}_4 : \quad U^2(r) = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2} + g^2 r^2, \quad h(r) = r. \quad (2.3)$$

Imposing BPS conditions leads to exactly two different possibilities in pure supergravity without vector multiplets, as analyzed long ago in [5]. One solution is usually referred to as "extreme RN-AdS electric solution", it is half-BPS and it requires $M = Q, P = 0$, hence

$$\text{extreme electric RN} - \text{AdS}_4 : \quad U^2(r) = \left(1 - \frac{Q}{r}\right)^2 + g^2 r^2, \quad h(r) = r. \quad (2.4)$$

The function $U(r)$ has no zeroes and therefore the spacetime has no horizon. The point $r = 0$ is then a naked singularity. The other solution is referred to as an "exotic AdS solution" and is only quarter-BPS, imposing $M = 0, P = 1/(2g)$,

$$\text{exotic AdS}_4 : \quad U^2(r) = \left(gr + \frac{1}{2gr}\right)^2 + \frac{Q^2}{r^2}, \quad h(r) = r. \quad (2.5)$$

This case has no flat space limit for $g \rightarrow 0$ and is therefore very different in behavior from the first solution. Still, the solution has a naked singularity.

The aim of this paper is to find a generalization of the second solution within $N = 2$ gauged supergravity with a number of vector multiplets such that this naked singularity is resolved due to non-trivial scalar behavior. We will focus on extending the exotic solution since the extension of the extreme RN-AdS solutions for many vector multiplets and non-trivial scalars has been investigated in [4] with the outcome of nakedly singular spacetimes once again. Some generalizations of the exotic solution also exist in the literature, e.g. in [12], but these set the scalars to constants and are thus not general enough to resolve the naked singularity. Our strategy will be to replace the cosmological constant with a nontrivial potential for the vector multiplet scalars that contains Fayet-Iliopoulos terms.

Anticipating our results, we now briefly explain how the exotic solution is modified to make proper black holes in AdS_4 . We set the electric charges to zero but allow for non-trivial scalars, which will in the end result in changing the metric function U to be²:

$$U^2(r) = \left(gr + \frac{c}{2gr}\right)^2, \quad (2.6)$$

²Here, the discussion is only schematic in order to underline the main point, the actual solution is more involved as we explain in sections 5- 7. There we also comment further on the other function in the metric, $h(r)$.

with a constant $c \neq 1$ that depends on the explicit running of the scalars. The important outcome from this is that in certain cases we will have $c < 0$, and then a horizon will appear at $r_h = \sqrt{\frac{-c}{2g^2}}$ to shield the singularity. In this way, one can find a static quarter-BPS asymptotically AdS₄ black hole with nontrivial scalar fields.

3 Gauged supergravity with Fayet-Iliopoulos parameters

In this work we focus on abelian gauged $N = 2$ supergravity in four dimensions in the absence of hypermultiplets. We consider n_V vector multiplets and keep the same conventions for metric signatures and field strenghts as in [13, 7]. For some background material on gauged $N = 2$ supergravity, see e.g. [14, 15, 16, 17]. As the gauge group is abelian, the vector multiplet scalars are neutral, and the only charged fields in the theory are the two gravitinos. This is usually referred to as Fayet-Iliopoulos (FI) gauging. The gauge fields that couple to the gravitinos appear in a linear combination of the graviphoton and the n_V vectors from the vectormultiplets, $\xi_\Lambda A_\mu^\Lambda$, with $\Lambda = 0, 1, \dots, n_V$. The constants ξ_Λ are called FI parameters³. The bosonic part of the Lagrangian for such a system is

$$\mathcal{L} = \frac{1}{2}R(g) + g_{i\bar{j}}\partial^\mu z^i \partial_\mu \bar{z}^{\bar{j}} + I_{\Lambda\Sigma} F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{2}R_{\Lambda\Sigma}\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma - g^2 V(z, \bar{z}) , \quad (3.1)$$

where

$$V = (g^{i\bar{j}} f_i^\Lambda \bar{f}_{\bar{j}}^\Sigma - 3\bar{L}^\Lambda L^\Sigma) \xi_\Lambda \xi_\Sigma \quad (3.2)$$

is the scalar potential. Here, the complex scalar fields z^i ($i = 1, \dots, n_V$) are expressed in terms of holomorphic symplectic sections $(X^\Lambda(z), F_\Lambda(z))$ (see [17] for a review), and the matrices $R_{\Lambda\Sigma}$ and $I_{\Lambda\Sigma}$ are the real and imaginary parts, respectively, of the period matrix defined by

$$\mathcal{N}_{\Lambda\Sigma} \equiv \begin{pmatrix} D_i F_\Lambda \\ \bar{F}_\Lambda \end{pmatrix} \cdot \begin{pmatrix} D_i X^\Sigma \\ \bar{X}^\Sigma \end{pmatrix}^{-1} , \quad (3.3)$$

with $D_i \equiv (\partial_i + \mathcal{K}_i)$ ⁴. The Kähler potential

$$\mathcal{K}(z, \bar{z}) = -\ln \left[i(\bar{X}^\Lambda(\bar{z}) F_\Lambda(z) - X^\Lambda(z) \bar{F}_\Lambda(\bar{z})) \right] \quad (3.4)$$

³The FI terms may also be understood from the triplet of quaternionic moment maps P_Λ^x in the absence of hypermultiplets. Using the local $SU(2)_R$ symmetry, we can rotate them such that $P_\Lambda^x = \delta^{x,3} \xi_\Lambda$, leaving a $U(1) \subset SU(2)_R$ as a residual symmetry. One often uses the terminology that this part of the R -symmetry group is gauged.

⁴More explicitly, the period matrix can be computed by

$$\mathcal{N}_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} + 2i \frac{\text{Im}(F_{\Lambda\Gamma}) X^\Gamma \text{Im}(F_{\Sigma\Delta}) X^\Delta}{X^\Gamma \text{Im}(F_{\Gamma\Delta}) X^\Delta} , \quad F_{\Gamma\Delta} \equiv \frac{\partial F_\Gamma}{\partial X^\Delta} .$$

determines the metric of the scalar field moduli space $g_{i\bar{j}} = \partial_{z^i} \partial_{\bar{z}^j} \mathcal{K} \equiv \mathcal{K}_{i\bar{j}}$. In case a prepotential exists, it is given by $F_\Lambda = \partial F / \partial X^\Lambda$, which we use in the examples discussed in section 7. We will further make use of the quantities

$$(L^\Lambda, M_\Lambda) \equiv e^{\mathcal{K}/2} (X^\Lambda, F_\Lambda) , \quad (f_i^\Lambda, h_{\Lambda,i}) \equiv e^{\mathcal{K}/2} (D_i X^\Lambda, D_i F_\Lambda) . \quad (3.5)$$

The supersymmetry variations for the gaugino and gravitino fields, respectively, are:

$$\delta_\varepsilon \lambda^{iA} = i \partial_\mu z^i \gamma^\mu \varepsilon^A - g^{i\bar{j}} \bar{f}_{\bar{j}}^\Lambda I_{\Lambda\Sigma} F_{\mu\nu}^{\Sigma-} \gamma^{\mu\nu} \epsilon^{AB} \varepsilon_B + i g g^{i\bar{j}} \bar{f}_{\bar{j}}^\Lambda \xi_\Lambda \sigma^{3,AB} \varepsilon_B , \quad (3.6)$$

$$\delta_\varepsilon \psi_{\mu A} = \nabla_\mu \varepsilon_A + 2i F_{\mu\nu}^{\Lambda-} I_{\Lambda\Sigma} L^\Sigma \gamma^\nu \epsilon_{AB} \varepsilon^B - \frac{g}{2} \sigma_{AB}^3 \xi_\Lambda L^\Lambda \gamma_\mu \varepsilon^B , \quad (3.7)$$

up to higher order terms in the fermions. This is sufficient for solutions where all fermions are set to zero. The upper index "–" on the fields strengths denotes their antiselfdual part. The supercovariant derivative of the spinor reads:

$$\nabla_\mu \varepsilon_A = (\partial_\mu - \frac{1}{4} \omega_\mu^{ab} \gamma_{ab}) \varepsilon_A + \frac{1}{4} (\mathcal{K}_i \partial_\mu z^i - \mathcal{K}_{\bar{i}} \partial_\mu \bar{z}^{\bar{i}}) \varepsilon_A + \frac{i}{2} g \xi_\Lambda A_\mu^\Lambda \sigma_A^{3B} \varepsilon_B , \quad (3.8)$$

and similarly for the gravitino's

$$\nabla_\mu \psi_{\nu A} = \partial_\mu \psi_{\nu A} + \dots + \frac{i}{2} g \xi_\Lambda A_\mu^\Lambda \sigma_A^{3B} \psi_{\nu B} . \quad (3.9)$$

The fact that only σ^3 appears in the supersymmetry transformation rules and covariant derivatives reflects the fact that the $SU(2)_R$ symmetry is broken to $U(1)$, as referred to in footnote 3.

We have to stress that the above theory is gauged only electrically, since we have used only electric fields A_μ^Λ for the gauging of the gravitino. Thus the FI parameters can be thought of as the electric charges $\pm e_\Lambda$ of the gravitino fields, with

$$e_\Lambda = g \xi_\Lambda , \quad (3.10)$$

The fact that the gravitinos have opposite electric charge finds its origin from the eigenvalues of σ^3 . Generically in such a theory one encounters a Dirac-like quantization condition in the presence of magnetic charges p^Λ ,

$$2e_\Lambda p^\Lambda = n , \quad n \in \mathbb{Z} , \quad (3.11)$$

as explained in more detail in [5]. Clearly, (3.11) is not a symplectic invariant, due to the choice of the gauging. Later, in section 6, we generalize this to include also magnetic gaugings.

4 Black hole ansatz and Killing spinors

As already stated in section 2, we look for a supersymmetric solution similar to the "exotic AdS solution" of [5], but with nonconstant scalar fields. We start with the general static metric ansatz

$$ds^2 = U^2(r) dt^2 - U^{-2}(r) dr^2 - h^2(r) (d\theta^2 + \sin^2 \theta d\varphi^2) , \quad (4.1)$$

and corresponding vielbein

$$e_\mu^a = \text{diag} \left(U(r), U^{-1}(r), h(r), h(r) \sin \theta \right) . \quad (4.2)$$

The non-vanishing components of the spin connection turn out to be:

$$\omega_t^{01} = U \partial_r U, \quad \omega_\theta^{12} = -U \partial_r h, \quad \omega_\varphi^{13} = -U \partial_r h \sin \theta, \quad \omega_\varphi^{23} = -\cos \theta . \quad (4.3)$$

We further assume that the gauge field strengths are given by

$$F_{tr}^\Lambda = 0, \quad F_{\theta\varphi}^\Lambda = \frac{p^\Lambda}{2} \sin \theta, \quad (4.4)$$

or alternatively

$$A_t^\Lambda = A_r^\Lambda = A_\theta^\Lambda = 0, \quad A_\varphi^\Lambda = -p^\Lambda \cos \theta, \quad (4.5)$$

which are needed in the BPS equations below. If we allow also electric charges, we then should use an electromagnetic basis $F_{\mu\nu}^\Lambda, G_{\mu\nu,\Lambda}$ ⁵, and require

$$G_{\Lambda,\theta\varphi} = \frac{q_\Lambda}{2} \sin \theta, \quad F_{\theta\varphi}^\Lambda = \frac{p^\Lambda}{2} \sin \theta . \quad (4.6)$$

These automatically solve the Maxwell equations and Bianchi identities in full analogy to the case of ungauged supergravity⁶ [18]. However, we start with a purely electric gauging (3.1) and we set the electric charges of the black hole to zero since otherwise we cannot directly solve for the gauge fields A_t^Λ that are needed for the BPS equations. This is a particular choice we make at this point in view of the BPS conditions we derive below. In section 6 we will explain how to explicitly find a solution also with electric charges in a more general electromagnetic gauging frame.

⁵The magnetic field strengths can be defined from the Lagrangian to be

$$G_{\Lambda\mu\nu} \equiv R_{\Lambda\Sigma} F_{\mu\nu}^\Sigma - \frac{1}{2} I_{\Lambda\Sigma} \epsilon_{\mu\nu\gamma\delta} F^{\Sigma\gamma\delta} .$$

⁶Notice that the vector field part of the Lagrangian (3.1) is the same as in the ungauged theory, so they have the same equations of motion. The only difference appears in the coupling to the gravitinos.

4.1 Killing spinor ansatz

With the gamma matrix conventions spelled out in Appendix A we make the following ansatz for the (chiral) Killing spinors:

$$\varepsilon_A = e^{i\alpha} \epsilon_{AB} \gamma^0 \varepsilon^B, \quad \varepsilon_A = \pm e^{i\alpha} \sigma_{AB}^3 \gamma^1 \varepsilon^B, \quad (4.7)$$

where α is an arbitrary constant phase, and the choice of sign in the second condition will lead to two distinguishable Killing spinor solutions with corresponding BPS equations. This Killing spinor ansatz corresponds (in our conventions for chiral spinors) to the Killing spinor projections derived in [5] for the exotic solutions. Note that the choice of phase α is irrelevant due to $U(1)_R$ symmetry, i.e. any value of α leads to the exact same physical solution. It will nevertheless amount to putting the symplectic sections of the vector multiplet moduli space in a particular frame, as we explain in more detail in the next subsection. Furthermore, from the above equations one can deduce that the Killing spinor can be parametrized as follows. Using our convention from App. A, one finds that, $\forall a \in \mathbb{C}$, for the upper sign (which we call type *I*) in (4.7):

$$\varepsilon_1^I = a(x) \begin{pmatrix} 1 \\ i \\ -i \\ -1 \end{pmatrix}, \quad \varepsilon_2^I = \bar{a}(x) e^{i\alpha} \begin{pmatrix} -i \\ 1 \\ 1 \\ -i \end{pmatrix}. \quad (4.8)$$

For the negative sign (type *II*) one finds,

$$\varepsilon_1^{II} = a(x) \begin{pmatrix} 1 \\ i \\ i \\ 1 \end{pmatrix}, \quad \varepsilon_2^{II} = \bar{a}(x) e^{i\alpha} \begin{pmatrix} i \\ -1 \\ 1 \\ -i \end{pmatrix}. \quad (4.9)$$

This type of Killing spinors explicitly break 3/4 of the supersymmetry. The two degrees of freedom of the complex function a give the remaining two supercharges.

We look for spacetimes that are static and spherically symmetric, so in particular invariant under the rotation group. This rotation group acts on spinors, and can in general leave or not leave our Killing spinor ansatz invariant. It will be a check on our explicit solution for the Killing spinors that they should be also rotationally invariant, just as in the original case for exotic solutions [5].

Note that our choice of Killing spinors makes them timelike, i.e. they give rise to a timelike Killing vector (see [19, 20] for more details about Killing spinor identities). One can then

show [7] that, to obtain a supersymmetric solution, one needs to check only the Maxwell equations and Bianchi identities in addition to the BPS conditions. The equations of motion for the other fields then follow, due to the timelike Killing spinor.

4.2 BPS conditions and attractor flow

With the above ansätze for the spacetime and the Killing spinors one can show that the gaugino and gravitino variations (3.6), (3.7) simplify substantially but do not yet vanish identically.

From the gaugino variation we obtain the following radial flow equations for the scalar fields:

$$e^{-i\alpha} U \partial_r z^i = g^{i\bar{j}} \bar{f}_{\bar{j}}^{\Lambda} \left(\frac{2I_{\Lambda\Sigma} p^{\Sigma}}{h^2} \mp g\xi_{\Lambda} \right), \quad (4.10)$$

where the two different signs correspond to the two types of Killing spinors in the given order.

If we require the gravitino variation (3.7) to vanish, we derive four extra equations that need to be satisfied (one for each spacetime index). The equations for t and θ determine the radial dependence of the metric components,

$$e^{i\alpha} \partial_r U = -\frac{2L^{\Lambda} I_{\Lambda\Sigma} p^{\Sigma}}{h^2} \pm g\xi_{\Lambda} L^{\Lambda}, \quad (4.11)$$

$$e^{i\alpha} \frac{U}{h} \partial_r h = \frac{2L^{\Lambda} I_{\Lambda\Sigma} p^{\Sigma}}{h^2} \pm g\xi_{\Lambda} L^{\Lambda}. \quad (4.12)$$

The φ component of the gravitino variation further constrains

$$2g\xi_{\Lambda} p^{\Lambda} = \mp 1, \quad (4.13)$$

and the radial part gives a differential equation for the Killing spinor, solved by

$$a(r) = a_0 \sqrt{U(r)} e^{-\frac{i}{2} \int A_r(r) dr}, \quad (4.14)$$

with

$$A_r(r) = -\frac{i}{2} \left(\mathcal{K}_i \partial_r z^i - \mathcal{K}_{\bar{j}} \partial_r \bar{z}^{\bar{j}} \right) \quad (4.15)$$

the $U(1)$ Kähler connection. These results are in agreement with rotational symmetry since the Killing spinor is only a function of r . The solution is 1/4 BPS and has two conserved supercharges, corresponding to the two free numbers of the complex constant a_0 . We further see that (5.8) does not give an extra constraint on the fields, but can be

used to determine the explicit radial dependence of the Killing spinor parameter $a(r)$. One can always evaluate the integral of $A(r)$ for a given solution and thus the Killing spinor can be explicitly found once the BPS equations (4.10)-(4.13) are satisfied.

Notice also that (4.13) is in accordance with the generalized Dirac quantization condition (3.11) with the smallest non-zero integer $n = \pm 1$. It will be interesting to understand how one can generate other solutions with higher values of n or whether supersymmetry always strictly constrains n as in the present case. Furthermore, it is easy to see that in the limit $g \rightarrow 0$ where the gauging vanishes one recovers the well-known first order attractor flow equations of black holes in ungauged $N = 2$ supergravity [8, 9, 10]. The presence of the extra terms due to the gauging is precisely where the difference between ungauged and gauged black holes lies. Thus we believe the BPS equations are now written in a simpler and more suggestive form compared to [1].

A short comment on the phase α is in order. One can see in eqs. (4.11), (4.12) that the quantities $e^{-i\alpha} L^\Lambda$ must always be real. Thus, if e.g. $\alpha = 0$ then L^Λ will need to be real, while if $\alpha = \frac{\pi}{2}$, L^Λ have to be imaginary. This $U(1)_R$ symmetry of the BPS conditions is of course well understood in the ungauged case and there are generally two ways of proceeding. One can just fix the phase to a particular value and go on to write down the solutions, as originally done in [18], or one can also put explicitly the phase factor in the definition of the sections as done in [21]. Here we choose to fix $\alpha = 0$ for the rest of the paper as it will minimize the factors of i in what follows (note that [18] makes the opposite choice and thus the solutions are given for the imaginary instead of the real parts of the sections). It should be clear how one can always plug back the factor of $e^{-i\alpha}$ and choose a different phase if needed in different conventions. In particular this choice implies that (after adding (4.11) and (4.12))

$$\xi_\Lambda \text{Im}(X^\Lambda) = 0 . \quad (4.16)$$

5 Black hole solutions

Now we would like to find explicit solutions to eqs. (4.10)-(4.12). We already know (by assumption) the solution for the vector field strengths (4.4), so we search for solutions of the metric functions $U(r), h(r)$ and the symplectic sections $X^\Lambda(r), F_\Lambda(r)$ that determine the scalars. We propose the following form for the solution of the BPS equations in the electric frame (for the choice of phase $\alpha = 0$):

$$\frac{1}{2} (X^\Lambda + \bar{X}^\Lambda) = H^\Lambda , \quad \frac{1}{2} (F_\Lambda + \bar{F}_\Lambda) = 0 , \quad (5.1)$$

$$H^\Lambda = \alpha^\Lambda + \frac{\beta^\Lambda}{r},$$

and

$$U(r) = e^{\mathcal{K}/2} \left(gr + \frac{c}{2gr} \right), \quad h(r) = r e^{-\mathcal{K}/2}, \quad (5.2)$$

where \mathcal{K} is the Kähler potential

$$e^{-\mathcal{K}} = i \left(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda \right), \quad (5.3)$$

and c some constant. The line element of the spacetime is then

$$ds^2 = e^\mathcal{K} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}} r^2 d\Omega_2^2. \quad (5.4)$$

The constant c above is not specified yet and depends explicitly on the chosen model. This is also the case for the constants $\alpha^\Lambda, \beta^\Lambda$ that may eventually be expressed in terms of the FI parameters ξ_Λ and the magnetic charges p^Λ . We give some explicit examples in section 7. Here we just use the above results to show how the BPS equations simplify to a form where they can be explicitly solved given a particular model with a prepotential (we further assume that (5.1) implies $\text{Im}(X^\Lambda) = 0$ in accordance with (4.16)). Eqs. (4.11)-(4.12), together with (5.1)-(5.2), lead to:

$$\xi_\Lambda \alpha^\Lambda = \pm 1, \quad \xi_\Lambda \beta^\Lambda = 0, \quad (5.5)$$

$$F_\Lambda \left(-2g^2 r \beta^\Lambda + c \alpha^\Lambda + 2gp^\Lambda \right) = 0. \quad (5.6)$$

Multiplying (4.10) with f_i^Λ we eventually obtain

$$\begin{aligned} \left(gr + \frac{c}{2gr} \right) \left(F_\Sigma X^\Sigma \partial_r X^\Lambda - X^\Lambda F_\Sigma \partial_r X^\Sigma \right) &= -\frac{1}{r^2} F_\Sigma \left(X^\Sigma p^\Lambda - X^\Lambda p^\Sigma \right) \\ &+ g F_\Sigma X^\Sigma \left(X^\Lambda \pm i F_\Pi X^\Pi (I^{-1})^{\Lambda\Gamma} \xi_\Gamma \right). \end{aligned} \quad (5.7)$$

We chose to rewrite it in this form in order to have equations only for the symplectic sections, as standardly done also in ungauged black holes literature [18]. In principle however f_i^Λ is non-invertible and thus (5.7) does not strictly speaking imply (4.10). Practically this never seems to be an issue since in fact (5.7) gives one extra equation. In all cases we solved explicitly the equations, we found that the condition coming from the gaugino variation is already automatically satisfied after solving (5.5) and (5.6). Unfortunately, we were not able to prove that it must vanish identically with the above ansatz.

Using (5.1) it is straightforward to prove that the Kähler connection (4.15) vanishes identically (c.f. Eq.(29) of [18]). Thus the functional dependence of the Killing spinors becomes

$$a(r) = \sqrt{U(r)} a_0, \quad (5.8)$$

just as in the original solution without scalars [5].

Note that with (5.1) one can now also show that the field strengths (4.4) identically solve the Bianchi identities and the Maxwell equation as they fall in the form (4.6) with $q_\Lambda = 0$. Thus any solution of (5.5)-(5.7) will be a supersymmetric solution of the theory with no further constraints.

One particular solution (the only one in absence of vector multiplets) of the above equations that is always present, is when $\alpha^\Lambda = -2gp^\Lambda$, $\beta^\Lambda = 0$, for all Λ , and $c = 1$. This solution is in fact the one discovered in [12] with constant scalars (X^Λ is constant when $\beta^\Lambda = 0$). However, this solution has a naked singularity, since $c > 0$. A horizon is not present in this case, since generally it will appear at $r_h^2 = -\frac{c}{2g^2}$ and thus only for $c < 0$. We will see in section 7 that indeed there exist solutions of the above equations in which $c < 0$, such that a proper horizon shields the singularity. These solutions however necessarily have nonzero β^Λ 's. Thus a proper black hole can only form in the presence of some sort of attractor mechanism for the scalar fields.

6 Black holes with electric and magnetic charges

We now explain how one can restore the broken electromagnetic duality invariance of the theory (3.1). As discussed in section 3, the electric gaugings break electromagnetic invariance, i.e. performing symplectic rotations leads us to a new Lagrangian that will be of different form from (3.1). One then needs to allow for both electric and magnetic gaugings and change the form of the scalar potential in order to recover the electromagnetic invariance of the ungauged theory. There have been various proposals in literature for extending it to gauged supergravity [22, 23]. It turns out that the correct approach to introducing real magnetic gaugings is the embedding tensor formalism, and we closely follow the analysis of [11]. It restores full electromagnetic duality invariance of the gauge theory (when the electric and magnetic charges are mutually local) by introducing additional tensor fields in the Lagrangian. Unfortunately the theory is not yet fully developed in general for supergravity (for rigid $N = 2$ supersymmetry, see [24]), but we will nevertheless be able to write down particular solutions due to the fact that we can do duality transformations on the solutions of the electrically gauged theory.

Even though we cannot give the most general Lagrangian and susy variations for the theory with electric and magnetic gaugings, we know how the bosonic part of the Lagrangian should look like in this very special case of FI gaugings. It is most instructive to integrate

out the additional tensor field that has to be introduced, following the procedure of section 5.1 of [11]. Exactly half of the gauge fields (we will originally have both electric and magnetic gauge fields, $(A_\mu^\Lambda, A_{\mu,\Lambda})$) will also be integrated out in this process. One first splits the index Λ in two parts, $\{\Lambda\} = \{\Lambda', \Lambda''\}$, for the non-vanishing electric and magnetic gauge fields respectively. The Lagrangian will then consist only of $A_\mu^{\Lambda'}, A_{\Lambda'',\mu}$, while $A_\mu^{\Lambda''}, A_{\Lambda',\mu}$ are integrated out together with the additional tensor field. Thus the linear combination of fields used for the $U(1)$ FI gauging is $\xi_{\Lambda'} A_\mu^{\Lambda'} - \xi^{\Lambda''} A_{\Lambda'',\mu}$. The $\xi^{\Lambda''}$'s are the magnetic charges of the gravitinos, and the new generalized Dirac quantization condition for electric and magnetic charges (q_Λ, p^Λ) of any solution is

$$2(e_{\Lambda'} p^{\Lambda'} - m^{\Lambda''} q_{\Lambda''}) = n, \quad n \in \mathbb{Z}, \quad (6.1)$$

with electric and magnetic gravitino charges $e_{\Lambda'} \equiv g\xi_{\Lambda'}$ and $m^{\Lambda''} \equiv g\xi^{\Lambda''}$. The scalar potential is then of the form

$$V = (g^{i\bar{j}} f_i^{\Lambda'} \bar{f}_{\bar{j}}^{\Sigma'} - 3\bar{L}^{\Lambda'} L^{\Sigma'}) \xi_{\Lambda'} \xi_{\Sigma'} - (g^{i\bar{j}} h_{i,\Lambda''} \bar{h}_{\bar{j},\Sigma''} - 3\bar{M}_{\Lambda''} M_{\Sigma''}) \xi^{\Lambda''} \xi^{\Sigma''}. \quad (6.2)$$

The main point about electromagnetic invariance is that the equations of motion are now invariant under the group $Sp(2(n_V + 1), \mathbb{R})$, which at the same time rotates the Lagrangian from a purely electric gauging frame to a more general electromagnetic gauging. The symplectic vectors transforming under the symmetry group are the sections (F_Λ, X^Λ) and the FI parameters $(\xi_\Lambda, \xi^\Lambda)$, as well as the vector field strengths $F_{\mu\nu}^\Lambda, G_{\mu\nu,\Lambda}$ (which come from the respective electric and magnetic gauge potentials $(A_\mu^\Lambda, A_{\mu,\Lambda})$). One can then see how natural equations (6.1),(6.2) are if we start from a purely electric frame with only $\xi_\Lambda, F_{\mu\nu}^\Lambda$ nonzero and then perform an arbitrary symplectic transformation. The important message is that once we have found a solution to the purely electric theory we can always perform any symplectic transformation of the theory to see how the solution looks like in a more general electromagnetic setting.

It is in fact easy to guess how the solution looks like in a more general theory with electric and magnetic gaugings. We have not proven the existence of such a BPS solution due to the lack of a properly defined Lagrangian and supersymmetry variations, but we can nevertheless indirectly find it by symplectic rotations. This procedure leads to a solution, where the metric is again given by (5.4), together with

$$\begin{aligned} F_{tr}^{\Lambda'} &= 0, & F_{\theta\varphi}^{\Lambda'} &= \frac{p^{\Lambda'}}{2} \sin \theta, \\ G_{\Lambda'',tr} &= 0, & G_{\Lambda'',\theta\varphi} &= \frac{q_{\Lambda''}}{2} \sin \theta, \end{aligned} \quad (6.3)$$

and harmonic functions that determine the sections

$$\begin{aligned}\frac{1}{2} \left(X^{\Lambda'} + \bar{X}^{\Lambda'} \right) &= H^{\Lambda'} , & \frac{1}{2} \left(F_{\Lambda'} + \bar{F}_{\Lambda'} \right) &= 0 , \\ \frac{1}{2} \left(X^{\Lambda''} + \bar{X}^{\Lambda''} \right) &= 0 , & \frac{1}{2} \left(F_{\Lambda''} + \bar{F}_{\Lambda''} \right) &= H_{\Lambda''} , \\ H^{\Lambda'} &= \alpha^{\Lambda'} + \frac{\beta^{\Lambda'}}{r} , & H_{\Lambda''} &= \alpha_{\Lambda''} + \frac{\beta_{\Lambda''}}{r} .\end{aligned}\tag{6.4}$$

The above should give solutions provided that the following identities (coming from the BPS conditions) are satisfied,

$$2g(\xi_{\Lambda'} p^{\Lambda'} - \xi^{\Lambda''} q_{\Lambda''}) = \mp 1 ,\tag{6.5}$$

$$\xi_{\Lambda'} \alpha^{\Lambda'} - \xi^{\Lambda''} \alpha_{\Lambda''} = \pm 1 , \quad \xi_{\Lambda'} \beta^{\Lambda'} - \xi^{\Lambda''} \beta_{\Lambda''} = 0 ,\tag{6.6}$$

$$F_{\Lambda'} \left(-2g^2 r \beta^{\Lambda'} + c \alpha^{\Lambda'} + 2g p^{\Lambda'} \right) - X^{\Lambda''} \left(-2g^2 r \beta_{\Lambda''} + c \alpha_{\Lambda''} + 2g q_{\Lambda''} \right) = 0 ,\tag{6.7}$$

together with the symplectic invariant version of (5.7) coming from contraction with f_i^{Λ} . This expression becomes lengthy and cumbersome to check and we will not write it down explicitly. In this case it will be easier to explicitly check the symplectic invariant version of (4.10) by first defining the complex vector multiplet scalars from the sections. Of course in case of confusion one can always take a model and rotate it to the electric frame where the susy variations are clearly spelled out ((3.6)-(3.7)).

7 Explicit black hole solutions

7.1 $n_V = 1$ with $F = -2i\sqrt{X^0(X^1)^3}$

This is the simplest prepotential in the ordinary electrically gauged theory that leads to a black hole solution. We have one vector multiplet with the prepotential

$$F = -2i\sqrt{X^0(X^1)^3} ,\tag{7.1}$$

thus one finds $X^0 = \alpha^0 + \frac{\beta^0}{r}$, $X^1 = \alpha^1 + \frac{\beta^1}{r}$ from (5.1). This theory exhibits an AdS_4 vacuum at the minimum of the scalar potential (corresponding to the cosmological constant)

$$V^* = \Lambda = -\frac{2g^2}{\sqrt{3}} \sqrt{\xi_0 \xi_1^3}\tag{7.2}$$

at $z^* = \sqrt{\frac{3\xi_0}{\xi_1}}$ (defining $z \equiv \frac{X^1}{X^0}$). This can be easily deduced using the results of [13]. Going through the BPS equations (5.5)-(5.6), we can fix all the constants of the solution

in terms of the FI parameters ξ_0, ξ_1 apart from one free parameter (here we leave β^1 to be free for convenience, but it can be traded for one of the magnetic charges or for β^0). We obtain that the magnetic charges are given by:

$$p^0 = \mp \frac{1}{g\xi_0} \left(\frac{1}{8} + \frac{8(g\xi_1\beta^1)^2}{3} \right), \quad p^1 = \mp \frac{1}{g\xi_1} \left(\frac{3}{8} - \frac{8(g\xi_1\beta^1)^2}{3} \right), \quad (7.3)$$

for spinor I and II respectively. The other constants in the solution are

$$\beta^0 = -\frac{\xi_1\beta^1}{\xi_0}, \quad \alpha^0 = \frac{\pm 1}{4\xi_0}, \quad \alpha^1 = \frac{\pm 3}{4\xi_1}, \quad c = 1 - \frac{32}{3}(g\xi_1\beta^1)^2. \quad (7.4)$$

Using the definition of the gravitino charges (3.10), $e_\Lambda = g\xi_\Lambda$, these relations imply

$$e_\Lambda \alpha^\Lambda = \pm g, \quad e_\Lambda \beta^\Lambda = 0, \quad 2e_\Lambda p^\Lambda = \mp 1, \quad (7.5)$$

and one can check that the complete solution is a function of the variables e_Λ, p^Λ and g . Note that in fact the dependence on g is artificial since it can always be absorbed in the definition of the coordinates. In particular, the rescaling $gr \rightarrow r, t \rightarrow gt$ makes the metric and the scalar flow dependent only on e_Λ, p^Λ as is also the form of the solution presented in [1].

Interestingly, one can verify that the condition coming from the gaugino variation, (5.7), is automatically satisfied with no further constraints. One can see that the two spinor types in the end amount to having opposite magnetic charges and to flipping some signs for the solution of the sections.

We now analyze the physical properties of the solution. In this case it is important to give explicitly the metric function in front of the dt^2 term. Using the form of the line element in (5.4), the specific form of the sections with constants given in (7.4), one can explicitly compute:

$$g_{tt} = \frac{2\sqrt{\xi_0\xi_1^3}r^2 \left(gr + \frac{1}{2gr} - \frac{16g}{3r}(\xi_1\beta^1)^2 \right)^2}{\sqrt{(r \mp 4\xi_1\beta^1)(3r \pm 4\xi_1\beta^1)^3}}. \quad (7.6)$$

The leading terms of the (infinite) asymptotic expansion of the metric for $r \rightarrow \infty$ are then

$$g_{tt}(r \rightarrow \infty) = -\frac{\Lambda r^2}{3} \left(1 + \frac{1}{2g^2}(1+c)\frac{1}{r^2} - \frac{256(\xi_1\beta^1)^3}{27}\frac{1}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) \right). \quad (7.7)$$

Clearly, the metric has the correct AdS_4 asymptotics. Although the constant term of the asymptotic expansion is not exactly 1 when we compare to the RN-AdS metric of section 2, we are still tempted to think that the coefficient in front of the $1/r$ term determines the physical mass of the black hole,

$$M = -\frac{128}{81}\Lambda(\xi_1\beta^1)^3. \quad (7.8)$$

The issue of defining the mass is a bit more subtle in asymptotically AdS spacetimes and we address it more carefully in section 9, where we verify our expectation.

One can also notice that there are some subtleties for the radial coordinate that usually do not appear for black hole spacetimes. In particular, $r = 0$ is neither a horizon (where $g_{tt} = 0$), nor a singularity (where $g_{tt} \rightarrow \infty$). In fact the point $r = 0$ is never part of the spacetime, since the singularity is always at a positive r , where the space should be cut off. Thus the r coordinate does not directly correspond to the radial coordinate from the singularity. The horizon for both signs is at

$$r_h = \sqrt{\frac{16}{3}(\xi_1\beta^1)^2 - \frac{1}{2g^2}} , \quad (7.9)$$

while genuine singularities will appear at $r_s = \pm 4\xi_1\beta^1, \mp \frac{4}{3}\xi_1\beta^1$. The spacetime will then continue only until the first singularity is encountered. If we want to have an actual black hole spacetime we must insist that the horizon shields the singularity, i.e. $r_h > r_s$, otherwise we again have a naked singularity and the sphere at r_h will not be part of the spacetime. This requirement further sets the constraints $|g\xi_1\beta^1| > \frac{3}{8}$, with $\xi_1\beta^1 < 0$ for solution I (upper sign) and $\xi_1\beta^1 > 0$ for solution II (lower sign). Since the parameter β^1 is at our disposal, it can always be chosen to be within the required range, thus the singularity can be shielded by a horizon in a particular parameter range for β^1 . So, putting together both solutions, we know that a proper black hole with a horizon will form in case $g\xi_1\beta^1 \in (-\infty, -\frac{3}{8}) \cup (\frac{3}{8}, \infty)$, with the corresponding relations given above between the magnetic charges and $\xi_1\beta^1$ for the two intervals. In between, we are dealing with naked singularities, which are of no interest for us at present. The constant c is always negative, and satisfies

$$c < -\frac{1}{2} , \quad (7.10)$$

which reflects again the existence of a horizon, as announced in section 2.

Let us now investigate further the properties of these new black holes. Their entropy is proportional to the area of the black hole at the horizon,

$$S = \frac{A}{4} = \frac{3}{4\Lambda} \sqrt{(r_h - r_{s,1})(r_h - r_{s,2})^3} = \frac{\sqrt{(r_h \mp 4\xi_1\beta^1)(3r_h \pm 4\xi_1\beta^1)^3}}{8\sqrt{\xi_0\xi_1^3}} , \quad (7.11)$$

so the entropy is effectively a function of ξ_0, ξ_1, β^1 , which can be rewritten in terms of the FI-terms and magnetic charges. Thus the entropy is a function of the black hole charges p^Λ and the gravitino charges e_Λ . One can further observe that in case of fixed gravitino charges e_Λ , the entropy scales quadratically with the parameter β^1 and thus linearly with the charges p^0 or p^1 in the limit of large charges. The opposite limit of fixed magnetic charges shows that the entropy remains constant for large gravitino charge.

It is interesting to note that the fact that the scalars at the horizon are fixed in terms of the gravitino and black hole charges is not directly obvious from the general form of the solution. The scalars depend on the constants $\alpha^\Lambda, \beta^\Lambda$ that might not always be fully determined by ξ_Λ, p^Λ . One example of this is for the prepotential $F = -iX^0X^1$ where the magnetic black hole charges are fully fixed in terms of FI parameters and either β^0 or β^1 can be freely chosen. However, one can show that in this case there is no parameter range for the β^Λ 's where the singularity is shielded by the horizon, thus black holes do not exist. In all the cases for which we checked that a black hole is possible we could verify that indeed the scalar values at the horizon can be expressed in terms of the charges and FI parameters, but we have no general proof of this⁷.

Another interesting question is what the near-horizon geometry of this black hole is. It is natural to expect that a static four dimensional BPS black hole has a near-horizon geometry of $AdS_2 \times S^2$ and this is indeed the case. The radii of the two spaces are

$$R_{S^2} = r_h e^{-\mathcal{K}/2}|_{r=r_h}, \quad R_{AdS_2} = \frac{e^{-\mathcal{K}/2}|_{r=r_h}}{2\sqrt{2}g}, \quad (7.12)$$

and it can be shown that $R_{S^2} > \sqrt{2}R_{AdS_2}$ from the constraints on having a horizon. As the radii are inversely proportional to the scalar curvature of these spaces, it follows that the overall $AdS_2 \times S^2$ space has a negative curvature, as expected for asymptotically AdS_4 black holes. Thus it is clear that near the horizon we do not observe a supersymmetry enhancement to a fully BPS vacuum as is the case for the asymptotically flat static BPS black holes⁸. Nevertheless, there could be a supersymmetry enhancement from a 1/4 BPS overall solution to a 1/2 BPS vacuum near the horizon.

7.2 $F = \frac{(X^1)^3}{X^0}$ in a mixed electromagnetic frame

In order to give an example of black hole solutions in a more general electromagnetic frame, one can rotate the sections and FI parameters of the previous example by the symplectic

⁷The BPS equations (5.5)-(5.7) can be relatively easily solved in full generality for a prepotential of the form $F = (X^0)^n(X^1)^{2-n}$. The outcome is that black holes exist for $n \in (0, 1)$. The solution for general n is in full analogy to the one presented here. There is only certain n dependence in the way the various constants depend on each other, which does not lead to any qualitative differences. Here we chose to explicitly describe the case with $n = 1/2$ since it is the most relevant case from a string theory point of view as we will see in the next section.

⁸ $AdS_2 \times S^2$ is maximally supersymmetric only for $R_{S^2} = R_{AdS_2}$ as shown in [13]

matrix

$$\mathcal{S} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \\ 0 & -3 & 0 & 0 \end{pmatrix}, \quad (7.13)$$

such that the prepotential after rotation corresponds to the well-studied in ungauged supergravity T^3 model with prepotential

$$F = \frac{(X^1)^3}{X^0}, \quad (7.14)$$

and the non-vanishing FI parameters are ξ_0, ξ^1 . The theory will then be electrically gauged with A_μ^0 and magnetically gauged with $A_{1,\mu}$. This prepotential cannot lead to an AdS BPS black hole in the purely electric gauging, because it does not exhibit a supersymmetric AdS₄ vacuum, as one can find using the methods of [13]. However, in this mixed electromagnetic gauging, the T^3 model does have a proper fully supersymmetric AdS vacuum.

Now we can follow the more general procedure outlined in section 6. In this case it turns out that $X^0 = \alpha^0 + \frac{\beta^0}{r}$, $F_1 = \alpha_1 + \frac{\beta_1}{r}$. The black hole solution will then have one magnetic charge p^0 and one electric charge q_1 . Going through the BPS equations (6.5)-(6.7), we can fix all the constants of the solution in terms of the FI parameters ξ_0, ξ^1 , apart from one free parameter which we choose to be β_1 . The charges are given by:

$$p^0 = \mp \frac{1}{g\xi_0} \left(\frac{1}{8} + \frac{8(g\xi^1\beta_1)^2}{3} \right), \quad q_1 = \pm \frac{1}{g\xi^1} \left(\frac{3}{8} - \frac{8(g\xi^1\beta_1)^2}{3} \right), \quad (7.15)$$

for spinor I and II respectively. The other constants in the solution are

$$\beta^0 = \frac{\xi^1\beta_1}{\xi_0}, \quad \alpha^0 = \pm \frac{1}{4\xi_0}, \quad \alpha_1 = \mp \frac{3}{4\xi^1}, \quad c = 1 - \frac{32}{3}(g\xi^1\beta_1)^2. \quad (7.16)$$

and one can see that the metric and scalar profile in this case are analogous to the example in the previous subsection, as expected. This confirms the consistency of the results in section 6. The entropy of the black hole is now a function of the electric and magnetic gravitino charges, $e_0 = g\xi_0$ and $m^1 = g\xi^1$, and the black hole charges p^0 and q_1 .

Note that we could have for instance rotated the frame from a fully electric to a fully magnetic frame, by the symplectic matrix

$$\mathcal{S} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1/3 \\ 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}, \quad (7.17)$$

and it turns out that the prepotential $F = -2i\sqrt{X^0(X^1)^3}$ is in fact invariant under this transformation. The resulting solution will be the same, but there will be two electric instead of two magnetic charges.

8 M-theory lift

An explicit string theory example of abelian gauged $N = 2$, $D = 4$ supergravity with FI terms was found by a consistent truncation of M-theory on S^7 in [25]. A standard Kaluza-Klein compactification on S^7 leads initially to an $SO(8)$ gauged $N = 8$ supergravity in four dimensions. To avoid some of the complications of non-abelian gauge fields, the authors of [25] further defined a consistent truncation of this theory to an $U(1)^4$ gauged $N = 2$ supergravity. The 11-dimensional metric ansatz is given by:

$$ds_{11}^2 = \Delta^{2/3} ds_4^2 + 2g^{-2} \Delta^{-1/3} \sum_{\Lambda=0}^3 a_{\Lambda}^{-1} \left(d\mu_{\Lambda}^2 + \mu_{\Lambda}^2 (d\phi_{\Lambda} + \frac{g}{\sqrt{2}} A^{\Lambda})^2 \right), \quad (8.1)$$

where $\Delta = \sum_{\Lambda} a_{\Lambda} \mu_{\Lambda}^2$ with the μ_{Λ} 's satisfying $\sum_{\Lambda} \mu_{\Lambda}^2 = 1$. They can be parameterized by the angles on the 3-sphere as explained in more detail in [25]. The remaining 4 angles ϕ_{Λ} together with the μ_{Λ} describe the internal space, while x^{μ} are coordinates of the four-dimensional spacetime on which the resulting $N = 2$, $D = 4$ gauged supergravity is defined. The factors a_{Λ} depend on the four-dimensional axio-dilaton scalars $\tau_i = e^{-\varphi_i} + i\chi_i$ (defined below) and the gauge fields $A^{\Lambda} = A_{\mu}^{\Lambda} dx^{\mu}$ are exactly the ones appearing in the four-dimensional theory. Note that if all the gauge fields are vanishing and the scalars are at the minimum of the potential, the internal space becomes exactly S^7 . Apart from the metric, the field strength of the 11-dimensional three form field is given by:

$$F_4 = \sqrt{2}g \sum_{\Lambda} (a_{\Lambda}^2 \mu_{\Lambda}^2 - \Delta a_{\Lambda}) \epsilon_4 + \frac{1}{\sqrt{2}g} \sum_{\Lambda} a_{\Lambda}^{-1} \bar{*} da_{\Lambda} \wedge d(\mu_{\Lambda}^2) - \frac{1}{g^2} \sum_{\Lambda} a_{\Lambda}^{-2} d(\mu_{\Lambda}^2) \wedge (d\phi_{\Lambda} + \frac{g}{\sqrt{2}} A^{\Lambda}) \wedge \bar{*} dA^{\Lambda}, \quad (8.2)$$

with $\bar{*}$ the Hodge dual with respect to the four-dimensional metric ds_4 , and ϵ_4 the corresponding volume form.

With these identifications, the four-dimensional $N = 2$ bosonic Lagrangian, written in our

conventions, reads

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}R(g) + \frac{1}{4} \sum_{i=1}^3 ((\partial\varphi_i)^2 + e^{2\varphi_i}(\partial\chi_i)^2) + Im(\mathcal{M})_{\Lambda\Sigma} F_{\mu\nu}^{\Lambda} F^{\Sigma\mu\nu} \\ & + \frac{1}{2} Re(\mathcal{M})_{\Lambda\Sigma} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^{\Lambda} F_{\rho\sigma}^{\Sigma} + 2g^2 \sum_{i=1}^3 \left(\cosh \varphi_i + \frac{1}{2} \chi_i^2 e^{\varphi_i} \right) . \end{aligned} \quad (8.3)$$

One can then check explicitly (using also the particular result for the matrix \mathcal{M} given in [25]) that the above Lagrangian is indeed of the form of (3.1) with prepotential

$$F = -2i\sqrt{X^0 X^1 X^2 X^3} , \quad (8.4)$$

where the sections X^{Λ} define the three scalars τ_i by $\frac{X^1}{X^0} \equiv \tau_2\tau_3$, $\frac{X^2}{X^0} \equiv \tau_1\tau_3$, $\frac{X^3}{X^0} \equiv \tau_1\tau_2$. The FI parameters take the particularly simple form

$$\xi_0 = \xi_1 = \xi_2 = \xi_3 = 1 . \quad (8.5)$$

In this theory one can find a black hole solution in analogy to the example in section 7.1. Following the general results in section 5, $X^{\Lambda} = \alpha^{\Lambda} + \frac{\beta^{\Lambda}}{r}$, and from (5.5)-(5.6) one can find the full solution with $\alpha^0 = \alpha^1 = \alpha^2 = \alpha^3 = \pm\frac{1}{4}$ and three arbitrary parameters $\beta^1, \beta^2, \beta^3$ (or equivalently p^1, p^2, p^3). We will not write down the full solution as the expressions for the constant c and the magnetic charges in terms of the β^{Λ} 's are very long and do not lead to further insight. It is clear that the particular solution when we choose $\beta^1 = \beta^2 = \beta^3$ in fact coincides precisely with the solution in section 7.1 and this means that in any case a genuine black hole of the M-theory reduction exists particularly when the three complex scalars are equal. In the full solution of course there is a wider range of values for $\beta^1, \beta^2, \beta^3$ that will lead to a black hole, but this will suffice for our purposes here.

We now comment on the meaning of these four-dimensional black holes from the point of view of M-theory as a first step towards constructing the corresponding microscopic theory. It is notable that the particular M-theory reduction we have leads to an electrically gauged $N = 2$ supergravity and thus the resulting solution has only magnetic charges. This in fact makes the higher dimensional interpretation a bit more involved. There are two main points one can notice about the full 11-dimensional geometry from the form in (8.1). First, due to the nonconstant scalars τ_i , the full space is a warped product of the internal seven-dimensional space with the AdS₄ black hole spacetime. Second, due to the non-vanishing gauge fields $A_{\varphi}^{\Lambda} = -p^{\Lambda} \cos \theta$, there is an explicit mixing between the four angles ϕ_{Λ} of the internal space and the four-dimensional angle φ . This leads to four topological charges of the 11-dimensional spacetime, in analogy to NUT charges. Note that in case the charges were only electric, i.e. $A_t^{\Lambda} = \frac{q^{\Lambda}}{r}$, the time coordinate would mix with the internal angles

and we would obtain four angular momenta, leading to the interpretation of the spacetime as arising from the decoupling limit of rotating M2-branes as explained in detail in [25]. In the present case however the interpretation of the four-dimensional black holes from M-theory is more involved because apart from M2-branes we need to have some Kaluza-Klein monopoles in the M-theory solution, in order to account for the topological charge coming from the magnetic charges in four dimensions. Unfortunately we were not able to find an explicit example for this type of solutions in the literature, which probably is also related to the fact that they would break almost all supersymmetry⁹.

9 Black hole mass

In eqn.(7.8) we proposed a formula for the black hole mass. In this section we provide more evidence for this using using holographic renormalization. The computation is in fact somewhat complicated due to the fact that it is hard to define an energy, respectively mass, for asymptotically AdS black holes with running scalars. A more detailed discussion on the complications due to the scalars can be found in [26]. The correct approach to the problem was developed in a series of papers [27], combining holographic regularization close to the AdS boundary with the Hamilton-Jacobi method for finding the appropriate counterterms. These results were collected by [28] in a form we can readily use for our purposes. For the particular class of black holes given by (5.4), we can apply the formulas of [28] and find the regularized energy to be

$$E_{reg} = -2\omega_2 \left(gr_0 + \frac{c}{2gr_0} \right)^2 r_0 \left(-\frac{r_0}{2} \mathcal{K}'(r_0) + 1 \right), \quad (9.1)$$

where ω_2 is the volume element of a unit 2-sphere and the cutoff r_0 has to be eventually taken to infinity. This expression clearly diverges, so one has to add to it the counterterm energy given by

$$E_{ct} = \omega_2 e^{-\mathcal{K}/2} \left(gr_0 + \frac{c}{2gr_0} \right) \left(r_0^2 W(\phi) + \frac{e^{\mathcal{K}}}{g} + \mathcal{O}(r_0^{-2}) \right). \quad (9.2)$$

The expression $W(\phi)$ requires some further explanation. It specifies the counterterms coming from the scalar fields and is referred to as "superpotential" due to its resemblance

⁹The black hole solutions in four dimensions preserve only two supercharges, i.e. they are 1/4 BPS in $N = 2$. In $N = 8$, they are 1/16 BPS. This means that at least 30 of the original 32 supercharges in the original 11-dimensional supergravity will have to be broken for the conjectured bound state of M2-branes and Kaluza-Klein monopoles.

with the usual meaning of superpotential in supergravity. It should be derived from the scalar potential via

$$V = 2G^{ij}(\phi)\frac{\partial W}{\partial\phi^i}\frac{\partial W}{\partial\phi^j} - \frac{3}{2}W^2. \quad (9.3)$$

However, this expression does not rely on any supersymmetry and is not necessarily unique as explained in more detail in [29] in the five-dimensional case. The important point is that one needs a set of real scalar fields ϕ^i which is not a priori the case in $N = 2$ supergravity. However, it might turn out in practice that the solution effectively truncates the real or imaginary part of the original complex scalars and thus one should be in principle able to find the superpotential. This is indeed what happens e.g. in the black hole solution coming from the $N = 8$ truncation described above. Due to the importance of this particular M-theory reduction, the theory was investigated and the corresponding superpotential already found in [28]. Let us first properly give the full solution in our conventions in order to be able to describe precisely the relation between mass and charges. We choose $X^1 = X^2 = X^3 = \alpha + \frac{\beta}{r}$ as explained in the previous section, and additionally have $X^0 = \alpha^0 - \frac{3\beta}{r}$. All the FI parameters are equal and set to one, thus the BPS equations result eventually in the following expression for the charges:

$$p^0 = \pm \frac{1}{g} \left(\frac{1}{4} - 48g^2\beta^2 \right), \quad p^1 = p^2 = p^3 = \mp \frac{1}{g} \left(\frac{1}{4} - 16g^2\beta^2 \right), \quad (9.4)$$

for spinor I and II respectively. The other constants in the solution are

$$\alpha^0 = \pm \frac{1}{4}, \quad \alpha = \pm \frac{1}{4}, \quad c = 1 - 96g^2\beta^2. \quad (9.5)$$

The horizon is then found at $r_h = \sqrt{48\beta^2 - \frac{1}{2g^2}}$ and requiring a genuine black hole with horizon constrains $g\beta \in (-\infty, -\frac{1}{8})$ for spinor I and $g\beta \in (\frac{1}{8}, +\infty)$ for spinor II. Again, we find that $c < -\frac{1}{2}$. One can compute the superpotential to be

$$W = \frac{g}{2} \left(\frac{(\pm r + 4\beta)^{3/4}}{(\pm r - 12\beta)^{3/4}} + 3 \frac{(\pm r - 12\beta)^{1/4}}{(\pm r + 4\beta)^{1/4}} \right). \quad (9.6)$$

Plugging this in (9.1)-(9.2) leads to

$$E_{ren} = (E_{reg} + E_{ct})_{r_0 \rightarrow \infty} = \mp \omega_2 (512g^2\beta^3), \quad (9.7)$$

so we can define the mass to be

$$M = \mp 512g^2\beta^3, \quad (9.8)$$

which is strictly bigger than zero for the black holes with horizon. In fact we obtain the following interesting relation after plugging in the possible ranges of β :

$$M > \frac{1}{g}. \quad (9.9)$$

This inequality seems to be generic enough independent of the technical details of the particular solution, so we expect that it holds in general for the new class of black holes.

It is very interesting to observe that the same value for the black hole mass can be derived in a straightforward way from the asymptotic expansion of the metric. In this case,

$$U^2(r \rightarrow \infty) = 4g^2r^2 + 4(1 - 48g^2\beta^2) \pm \frac{1024g^2\beta^3}{r} + \mathcal{O}\left(\frac{1}{r^2}\right). \quad (9.10)$$

Assuming that the coefficient in front of $1/r$ is indeed $-2M$ as in (2.3), we get back the same expression for the mass, (9.8). This is a nice independent confirmation that the procedure of holographic renormalization is well-defined.

As already shown in [27, 28], the standard laws of black hole thermodynamics hold with the above definition of holographically renormalized energy. We can then summarize that the new black hole solutions behave quite differently than the usual case. All physical parameters of the solution are fixed in terms of the gauge coupling g and the constants β (that can be related to the charges). The solutions are singular in the limits $g \rightarrow 0$ and $g \rightarrow \infty$. The limit of large charges corresponds to large β and this will be the parameter that is easier to work with. Schematically, the physical parameters in the large β limit for fixed g (i.e. fixed AdS_4 radius) are

$$(r_h - r_s) \sim \beta, \quad p \sim \beta^2, \quad M \sim \beta^3, \quad S \sim \beta^2, \quad (9.11)$$

with $r_h - r_s$ the radius of the black hole. It is then clear that the entropy in fact scales linearly with the charges, while the mass scales as $p^{3/2}$. This behavior is very atypical for black holes and it would be interesting to justify it on more general grounds from the supersymmetry algebra in AdS.

10 Outlook

From this work it should be clear that one implicit assumption about solutions in gauged supergravities is in fact incorrect. There do exist qualitatively very different types of space-time solutions in gauged supergravity with vector multiplets compared to the minimally gauged supergravity case. As examples, we discussed supersymmetric, static AdS_4 black holes with spherical symmetry in gauged supergravity with Fayet-Iliopoulos terms. To achieve a full classification of black holes in gauged supergravity, one has to consider a general supergravity setup with arbitrary number of vector multiplets and also potentially

hypermultiplets and tensor multiplets. The present work is in this respect a small step towards a broader understanding of such black hole solutions.

Further it is clear that the solutions described here are a very particular type and one can imagine different extensions to, e.g., rotating 1/2 and 1/4 BPS black holes with nontrivial scalars along with higher dimensional analogues of the static solution. The role and exact meaning of the attractor mechanism in AdS₄ black holes must also be better understood, an issue related with the construction of M-brane or D-brane description of these black holes. In this sense it is important to understand clearly the physical reason why the entropy of the black holes depends also on the gravitino charges. The thermodynamic description and the precise BPS bounds also need a more solid basis. It will be interesting to extend the present solutions also to extremal non-BPS and finite temperature analogues. We hope to address at least some of these issues in the future.

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A Gamma matrix conventions

The Dirac gamma-matrices satisfy

$$\begin{aligned}\{\gamma_a, \gamma_b\} &= 2\eta_{ab} , \\ [\gamma_a, \gamma_b] &\equiv 2\gamma_{ab} , \\ \gamma_5 &\equiv -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\gamma^0\gamma^1\gamma^2\gamma^3 .\end{aligned}\tag{A.1}$$

In addition, they can be chosen such that

$$\gamma_0^\dagger = \gamma_0, \quad \gamma_0\gamma_i^\dagger\gamma_0 = \gamma_i, \quad \gamma_5^\dagger = \gamma_5, \quad \gamma_\mu^* = -\gamma_\mu .\tag{A.2}$$

An explicit realization of such gamma matrices is the Majorana basis, given by

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}, \quad (\text{A.3})$$

where the $\sigma^i; i = 1, 2, 3$ are the Pauli matrices. Their $SU(2)$ matrix indices A, B can be lowered or raised with the antisymmetric tensor. We then obtain the following set of matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \text{indices}_A{}^B. \quad (\text{A.4})$$

$$\sigma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \text{indices}_{AB}. \quad (\text{A.5})$$

$$\sigma^1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \text{indices}^{AB}. \quad (\text{A.6})$$

Notice the property $(\sigma_{AB}^i)^* = -\sigma^{iAB}$.

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